Hw 4 Solution

Problem 2.4.1 Solution

Using the CDF given in the problem statement we find that

(a) $P[Y < 1] = 0$

(b) $P[Y \leq 1] = 1/4$

(c) $P[Y > 2] = 1 - P[Y \leq 2] = 1 - 1/2 = 1/2$

(d) $P[Y \geq 2] = 1 - P[Y < 2] = 1 - 1/4 = 3/4$

(e) $P[Y = 1] = 1/4$

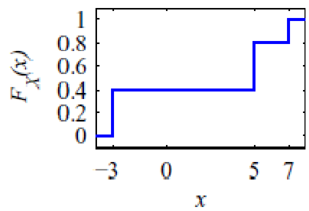
(f) $P[Y = 3] = 1/2$

(g) From the staircase CDF of Problem 2.4.1, we see that Y is a discrete random variable. The jumps in the CDF occur at at the values that Y can take on. The height of each jump equals the probability of that value. The PMF of Y is

$$P_Y(y) = \begin{cases} 1/4 & y = 1 \\ 1/4 & y = 2 \\ 1/2 & y = 3 \\ 0 & \text{otherwise} \end{cases} \qquad (1)$$

Problem 2.4.3 Solution

(a) Similar to the previous problem, the graph of the CDF is shown below.



$$F_X(x) = \begin{cases} 0 & x < -3 \\ 0.4 & -3 \leq x < 5 \\ 0.8 & 5 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$

(b) The corresponding PMF of X is

$$P_X(x) = \begin{cases} 0.4 & x = -3 \\ 0.4 & x = 5 \\ 0.2 & x = 7 \\ 0 & \text{otherwise} \end{cases}$$

Problem 2.5.2 Solution

Voice calls and data calls each cost 20 cents and 30 cents respectively. Furthermore the respective probabilities of each type of call are 0.6 and 0.4.

(a) Since each call is either a voice or data call, the cost of one call can only take the two values associated with the cost of each type of call. Therefore the PMF of X is

$$P_X(x) = \begin{cases} 0.6 & x = 20 \\ 0.4 & x = 30 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

(b) The expected cost, $E[C]$, is simply the sum of the cost of each type of call multiplied by the probability of such a call occurring.

$$E[C] = 20(0.6) + 30(0.4) = 24 \text{ cents} \tag{2}$$

Problem 2.5.8 Solution

The following experiments are based on a common model of packet transmissions in data networks. In these networks, each data packet contains a cylic redundancy check (CRC) code that permits the receiver to determine whether the packet was decoded correctly. In the following, we assume that a packet is corrupted with probability $\epsilon = 0.001$, independent of whether any other packet is corrupted.

(a) Let $X = 1$ if a data packet is decoded correctly; otherwise $X = 0$. Random variable X is a Bernoulli random variable with PMF

$$P_X(x) = \begin{cases} 0.001 & x = 0 \\ 0.999 & x = 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The parameter $\epsilon = 0.001$ is the probability a packet is corrupted. The expected value of X is

$$E[X] = 1 - \epsilon = 0.999 \tag{2}$$

(b) Let Y denote the number of packets received in error out of 100 packets transmitted. Y has the binomial PMF

$$P_Y(y) = \begin{cases} \binom{100}{y}(0.001)^y(0.999)^{100-y} & y = 0, 1, \ldots, 100 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The expected value of Y is

$$E[Y] = 100\epsilon = 0.1 \tag{4}$$

(d) If packet arrivals obey a Poisson model with an average arrival rate of 1000 packets per second, then the number N of packets that arrive in 5 seconds has the Poisson PMF

$$P_N(n) = \begin{cases} 5000^n e^{-5000}/n! & n = 0, 1, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The expected value of N is $E[N] = 5000$.

Problem 2.8.1 Solution

Given the following PMF

$$P_N(n) = \begin{cases} 0.2 & n = 0 \\ 0.7 & n = 1 \\ 0.1 & n = 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) $E[N] = (0.2)0 + (0.7)1 + (0.1)2 = 0.9$

(b) $E[N^2] = (0.2)0^2 + (0.7)1^2 + (0.1)2^2 = 1.1$

(c) $\text{Var}[N] = E[N^2] - E[N]^2 = 1.1 - (0.9)^2 = 0.29$

(d) $\sigma_N = \sqrt{\text{Var}[N]} = \sqrt{0.29}$